Belief Logic Programming with Cyclic Dependencies

3rd International Conference on Web Reasoning and Rule Systems

Hui Wan

Stony Brook University, New York

Uncertainty in Rule-based KBs

- Formalisms for representing uncertainty
  - Probabilities
  - Fuzzy sets
  - Belief functions

- *Our approach*: based on belief functions

Belief Concept in Everyday Life

- “Based on the tests, the doctor is 60% sure that Tim has cancer.”
- “Look, the ground is wet, I can say with 0.7 certainty that it rained last night.”
- “Based on the evidence, we believe with the confidence of 0.99 that O.J. Simpson committed the murder.”

These are not random events.

Degrees of belief, not probabilities.

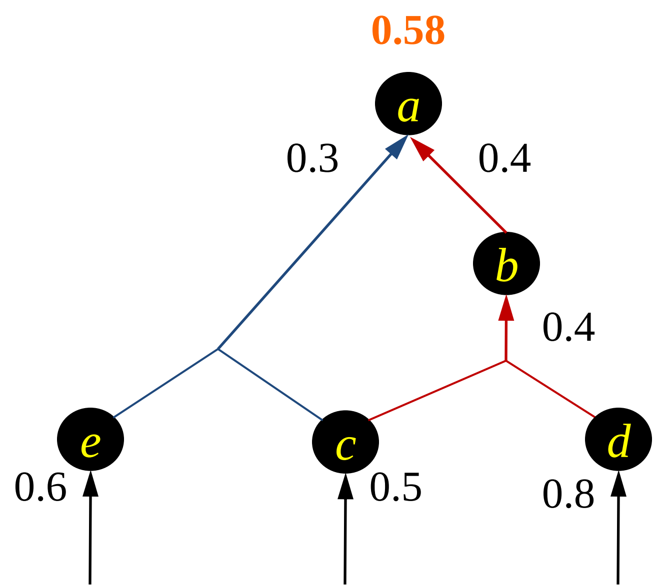
Motivation:
Evidence Combination in Rule-based KBs

- What is the belief in a?

$$\begin{array}{ll} & a \text{ :- } b. \\ & a \text{ :- } c \wedge e. \\ & b \text{ :- } c \wedge d. \\ 0.5 & c. \\ 0.8 & d. \\ 0.6 & e. \end{array}$$

$\Phi(x,y) = x+y-xy$ — combination function

$\Phi(0.3,0.4) =$ **0.58**



- But inferences for a are ***correlated*** so should not be combined outright.
  - Use combination function "max"?
    - No entailment between the two pieces of evidence.
  - The belief in a should be < 0.58, and > 0.4. - How much?

- Things become even more complicated when rules are uncertain.

- Goal
  - Theory for combining evidence from sources that might be *correlated*
- Prior work
  - Does not account for structural correlation through rules (as in the example)
- Our prior work: Belief LP (LPNMR09, SUM09)
  - solves this problem, but no ground recursion is allowed

Syntax

$$[v, w] \quad Head :- \ Body$$

belief factor, $0 \leq v \leq w \leq 1$
e.g., [0.2, 0.4]

an atom
e.g., $p(X)$

Conjunction of literals,
e.g., $p(X) \wedge b \wedge \bar{c}$

Our prior work:

no cyclic dependencies among atoms

[0.6,0.8] a :- $b \wedge c$

[0.3,0.9] b :- $d \wedge \bar{e}$

[0.8,1] d :- $a \wedge f$

Meaning of Rule

$$[v, w] \quad X :- \ Body$$

- If $Body$ holds, then this rule supports
  - X to the degree of v
  - $\overline{X}$ to the degree of $1 - w$

- The semantics ensures that

  if only one rule supports X

$$\frac{Bel(X \wedge Body)}{Bel(Body)} = v \qquad \frac{Bel(\overline{X} \wedge Body)}{Bel(Body)} = 1 - w$$

  Similar intuition holds for multiple rules

  (with appropriate modifications)

Combination Functions

- **Combination function**:

$$\Phi(\{\ [v_1\,,\,w_1],\ \ldots,\ [v_n\,,\,w_n]\ \}) = [v,\,w]$$

multiset of belief factors

combined belief factor

  - $\Phi(\{\ [v,\,w]\ \}) = [v,\,w]\ ,\ \Phi(\{\}) = [0,\,1]$

- Each atom X has an associated combo function Φ_X
  - Φ_X decides how to combine beliefs in X
  - Different Φ_X for different application domains
  - Examples: Dempster's rule, *max*, …

Semantics: Basics

notion	example
Herbrand base B_P	$B_P=\{a, b, c\}$
Truth valuation $I\colon\ B_P \rightarrow \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$ *tval*(***P***) : the set of all truth valuations over B_P	$I_1(a)=\mathbf{t},\ I_1(b)=\mathbf{t},\ I_1(c)=\mathbf{t}$ $I_2(a)=\mathbf{t},\ I_2(b)=\mathbf{t},\ I_2(c)=\mathbf{f}$ $I_3(a)=\mathbf{t},\ I_3(b)=\mathbf{t},\ I_3(c)=\mathbf{u}$ … $I_{27}(a)=\mathbf{u},\ I_{27}(b)=\mathbf{u},\ I_{27}(c)=\mathbf{u}$
Support function $m\colon\ tval(\boldsymbol{P}) \rightarrow [0,1]$ s.t. $\sum_{I \in tval(P)} m(I) = 1$	$m(I_1)= 0.5$ $m(I_3)= 0.5$ $m(I_k)= 0$, if $k \neq 1,\ k \neq 3$
Belief function *bel* $bel(F) = \sum_{I \models F} m(I)$ F: a boolean formula $\models$: entailment in Lukasiewicz 3-valued logic	

Plays the role of interpretation in classical LP.

Semantics

- Model of a belief logic program:
  - a belief function

- Properties
  - Non-monotonic
  - Strong relation to paraconsistent reasoning and defeasible reasoning

Declarative Semantics

- The set of rules that support X and fire in I
  $\boldsymbol{P}_I(X)$ $=\{R \mid R \in \boldsymbol{P}, X \in \mathrm{Head}(R),\ I \models \mathrm{Body}(R)\}$

- $s_{\boldsymbol{P}}(I,X)$: the support in I for X

– If $\boldsymbol{P}_I(X)=\{\}$,

$$s_P(I,X)=\begin{cases} 1 & \text{if } I(X)=u \\ 0 & \text{if } I(X)=t \text{ or } I(X)=f \end{cases}$$

– If $\boldsymbol{P}_I(X)=\{R_1,\ldots,R_k\}$,
  let $[v,w]$ be the result of applying Φ_X to the belief factors of $R_1,\ldots,R_k$

$$s_P(I,X)=\begin{cases} v & \text{if } I(X)=t \\ 1-w & \text{if } I(X)=f \\ w-v & \text{if } I(X)=u \end{cases}$$

- The support in I for $\boldsymbol{P}$ as a whole $\hat{m}_P(I)=\prod_{X\in B_P} s_P(I,X)$

- Model for $\boldsymbol{P}$:
  A belief function *bel*: $F \rightarrow \sum_{I\in Tval(B_P), I\models F} \hat{m}_P(I)$

  a boolean formula

This Paper

- Method to deal with arbitrary belief logic programs, including the cyclic ones
  - transformation-based
  - has good properties
  - circular feedback is not problematic

Cyclic Program

[0.8, 1] *disease*(*X*) :- *test_pos*(*X*).

// Positive test result supports the diagnosis of a person having the disease.

[0.6, 1] *disease*(*X*) :- *contact*(*X*, *Y*) /\ *disease*(*Y*).

// The disease is contagious.

[1, 1] *test_pos*(p3).

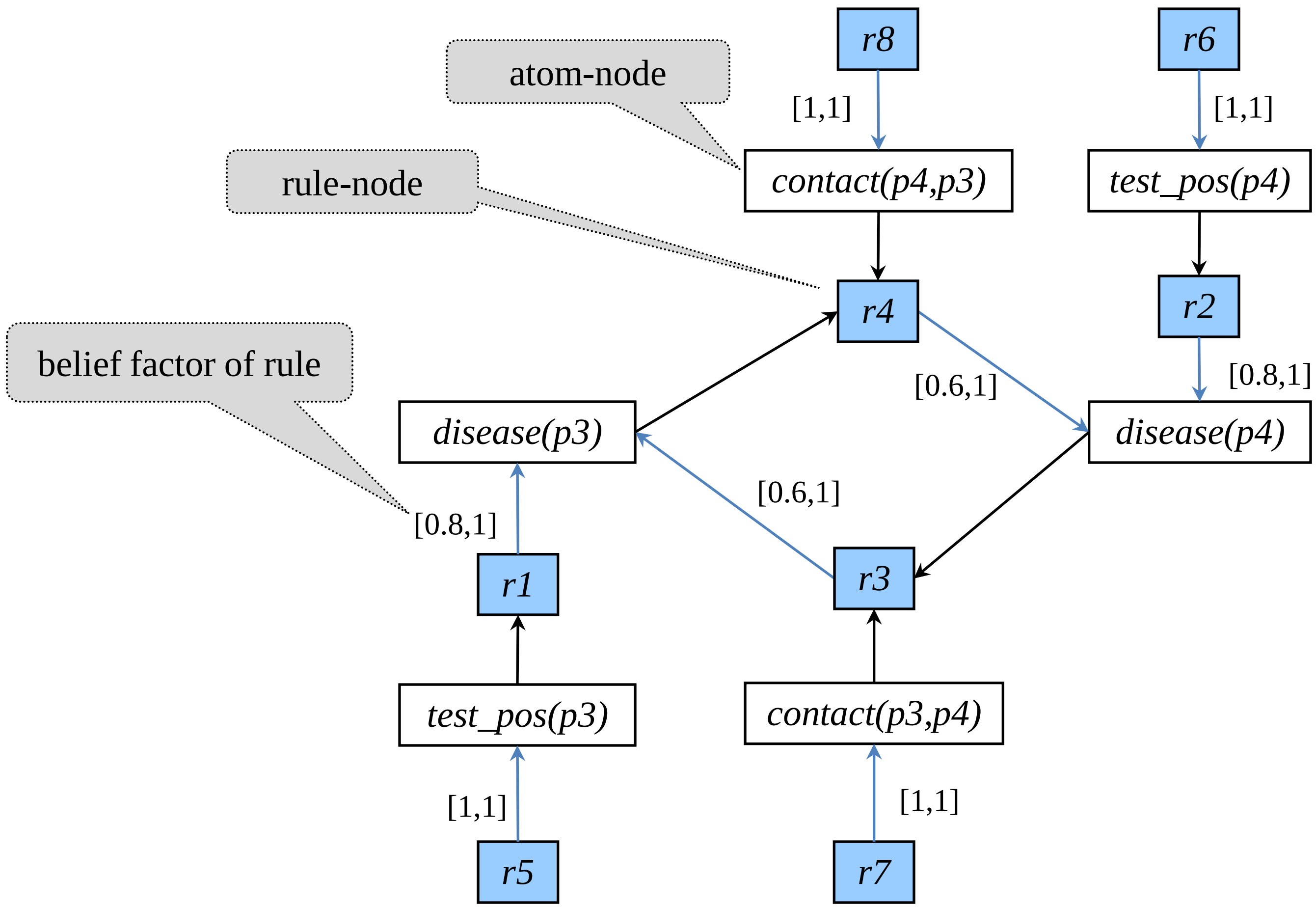
[1, 1] *contact*(p3, p4).

[1, 1] *test_pos*(p4).

[1, 1] *contact*(p4, p3).

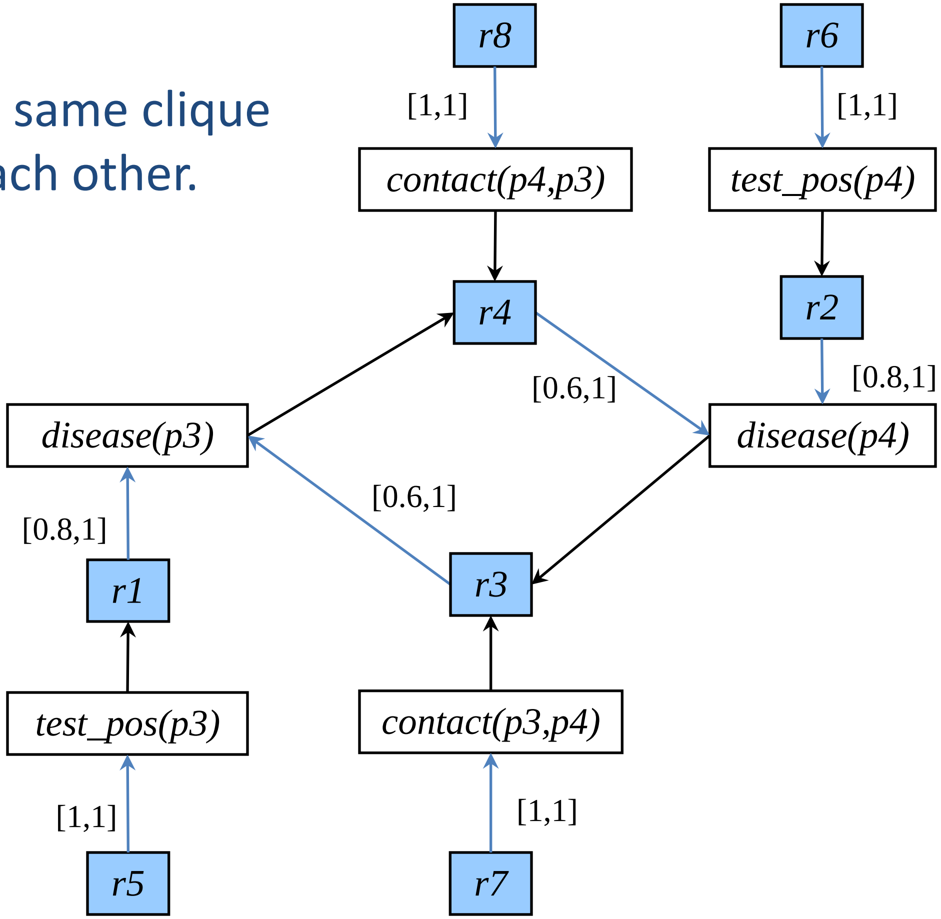
r1	[0.8, 1]	*disease*(p3) :- *test_pos*(p3).
r2	[0.8, 1]	*disease*(p4) :- *test_pos*(p4).
r3	[0.6, 1]	*disease*(p3) :- *contact*(p3, p4) /\ *disease*(p4).
r4	[0.6, 1]	*disease*(p4) :- *contact*(p4, p3) /\ *disease*(p3).
r5	[1, 1]	*test_pos*(p3).
r6	[1, 1]	*test_pos*(p4).
r7	[1, 1]	*contact*(p3, p4).
r8	[1, 1]	*contact*(p4, p3).

Dependency Graph



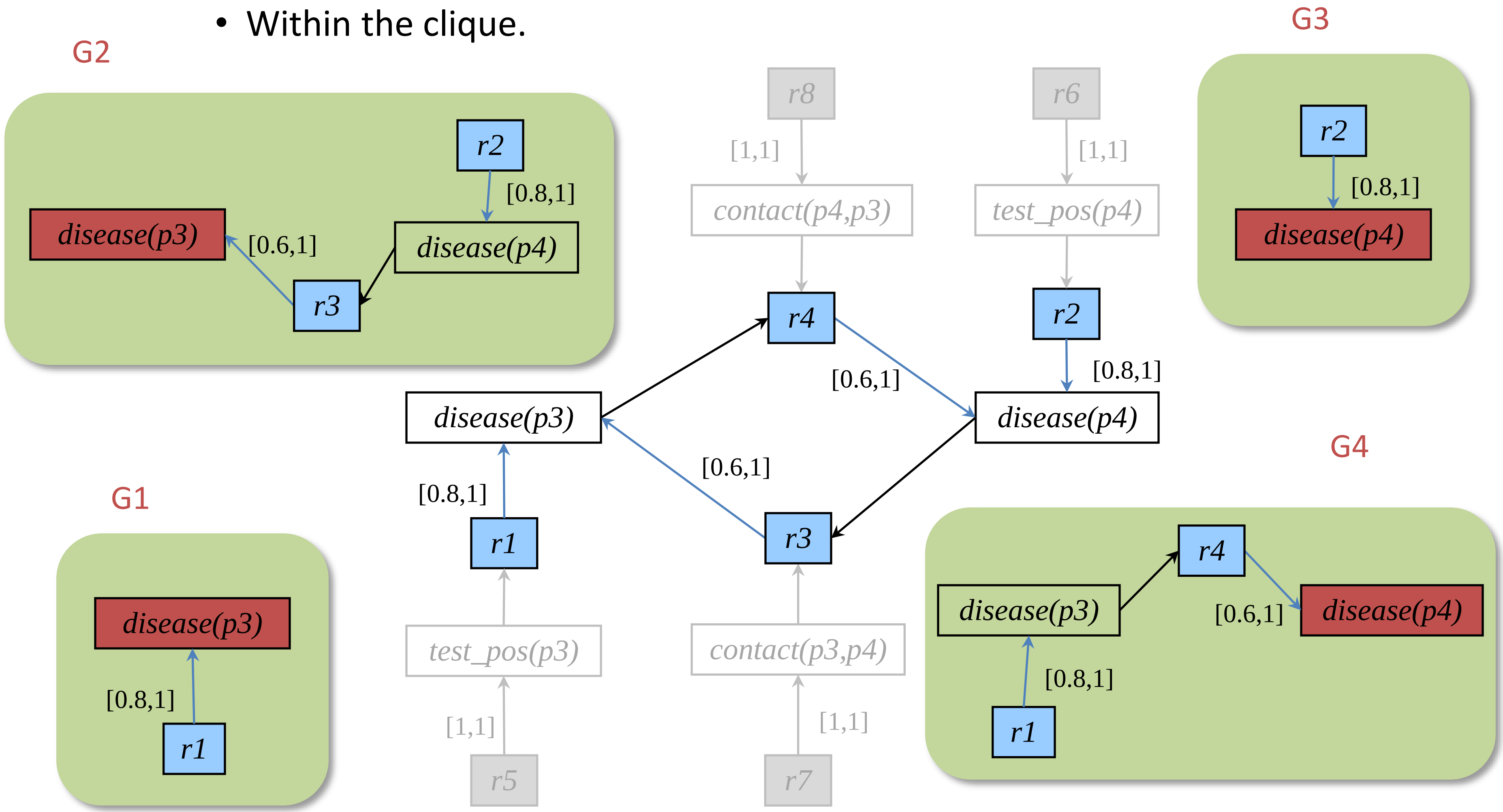
Atom Cliques

Two atoms are in the same clique iff they depend on each other.

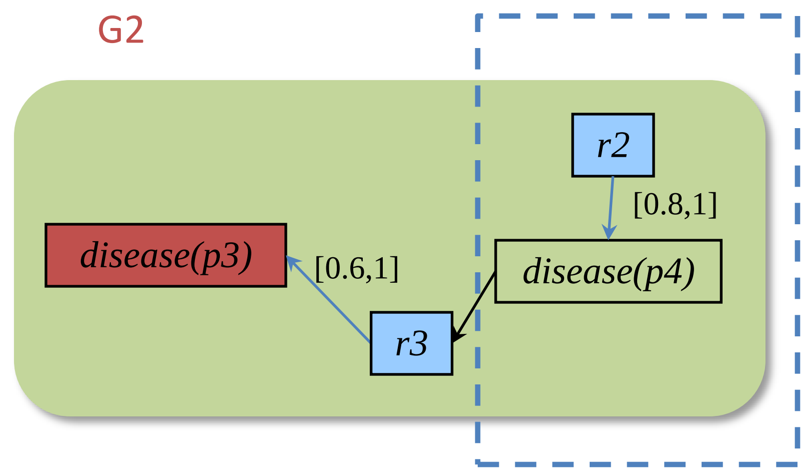


Partial Proof DAG (pp-DAG)

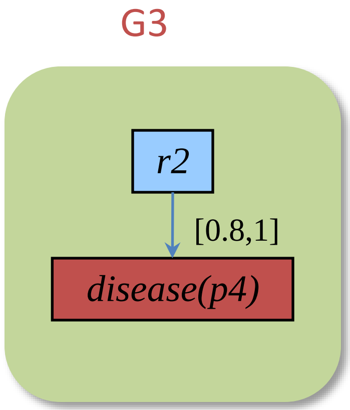
- Corresponds to an SLD derivation of the query.
- No cycles allowed.
- Within the clique.

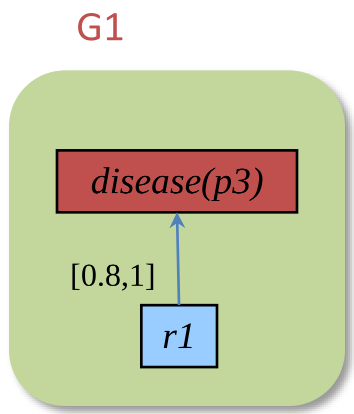


Child pp-DAG

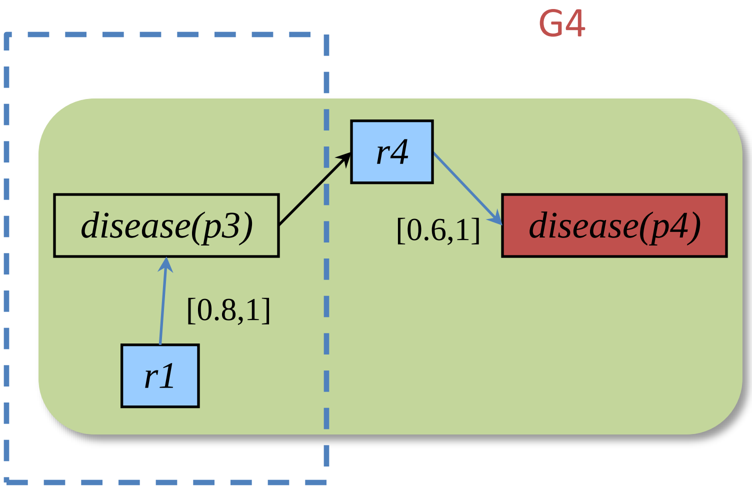


G3 is a child pp-DAG of G2.





G1 is a child pp-DAG of G4.



Declyclification

- For every rule whose head atom is in a cycle

$$R \quad [v,w] \quad A_0 \ :\text{-}\ A_1 \wedge \ \ldots \wedge A_k \wedge \overline{A_{k+1}} \wedge \ \ldots \wedge \overline{A_n} \wedge Conj.$$

where $A_0, \ldots A_n$ are in the same clique, and no atom in $Conj$ is in this clique.

  - Replace it with $[v,w] \quad A_0 \ :\text{-}\ R.$
  - For every list of pp-DAGs, $G_0, \ldots, G_n$, such that
    - A_i is G_i's root, and R is in G_0
    - $G_1, \ldots, G_n$ are children of G_0

  add rules

$$[v,w] \quad A_0^{G_0} \ :\text{-}\ A_1^{G_1} \wedge \ \ldots \wedge A_k^{G_k} \wedge \overline{A_{k+1}^{G_{k+1}}} \wedge \ \ldots \wedge \overline{A_n^{G_n}} \wedge Conj.$$

$$[1,1] \quad R \ :\text{-}\ A_1^{G_1} \wedge \ \ldots \wedge A_k^{G_k} \wedge \overline{A_{k+1}^{G_{k+1}}} \wedge \ \ldots \wedge \overline{A_n^{G_n}} \wedge Conj.$$

- Intuition:
  - The belief in A^G is precisely that part of the belief in A, which is justified by the derivation that corresponds to G.
  - The belief in R is the belief in the rule body being derived without any loop influence.
  - The belief in A_0 is the combination of the support to A_0 from all the rules with A_0 in head.

P

r1	[0.8, 1]	$disease(p3)$:- $test\_pos(p3)$.
r2	[0.8, 1]	$disease(p4)$:- $test\_pos(p4)$.
r3	[0.6, 1]	$disease(p3)$:- $contact(p3, p4) \wedge disease(p4)$.
r4	[0.6, 1]	$disease(p4)$:- $contact(p4, p3) \wedge disease(p3)$.
...		

acyclic(P)

[0.8, 1]	$disease(p3)$	:-	$r1$.
[1, 1]	$r1$	:-	$test\_pos(p3)$.
[0.8, 1]	$disease(p4)$	:-	$r2$.
[1, 1]	$r2$	:-	$test\_pos(p4)$.
[0.6, 1]	$disease(p3)$	:-	$r3$.
[1, 1]	$r3$	:-	$contact(p3, p4) \wedge disease^{G2}(p4)$.
[0.6, 1]	$disease(p4)$	:-	$r4$.
[1, 1]	$r4$	:-	$contact(p4, p3) \wedge disease^{G1}(p3)$.
[0.8, 1]	$disease^{G1}(p3)$	:-	$test\_pos(p3)$.
[0.8, 1]	$disease^{G2}(p4)$	:-	$test\_pos(p4)$.
[0.6, 1]	$disease^{G3}(p3)$	:-	$contact(p3, p4) \wedge disease^{G2}(p4)$.
[0.6, 1]	$disease^{G4}(p4)$	:-	$contact(p4, p3) \wedge disease^{G1}(p3)$.
…			

$bel(disease(p3))=0.896$

$bel(disease(p4))=0.896$

G1, G2 contribute to disease(p3).
G3, G4 contribute to disease(p4).
G3 is a child pp-DAG of G2.
G1 is a child pp-DAG of G4.

- The model of a general program P is the model of $acyclic(P)$.

- Properties
  - Backward compatible with acyclic semantics
  - Circular feedback is not problematic

- Transformational semantics
  - based on a global decyclification

- Fixpoint semantics
  - based on a sequence of local decyclifications

- Equivalent results

A step in the fixpoint computation:

- Consider: the rules with head in an atom clique C
- Already known: the support to truth valuation I for the atoms which C depends on

[0.8, 1]	a :- d .
[0.8, 1]	b :- e .
[0.6, 1]	a :- $b \wedge f1$.
[0.6, 1]	b :- $a \wedge f2$.
[1, 1]	d .
[1, 1]	e .
[0.5, 0.5]	$f1$.
[0.5, 1]	$f2$:- $f1$.

I

$f1$	$f2$	d	e
t	**t**	**t**	**t**

$m(I)$=0.25

[0.8, 1]	a ~~:- d~~.
[0.8, 1]	b ~~:- e~~.
[0.6, 1]	a :- b ~~$\wedge f1$~~ .
[0.6, 1]	b :- a ~~$\wedge f2$~~ .

[0.8, 1]	a :- $r1$.
[1, 1]	$r1$.
[0.8, 1]	b :- $r2$.
[1, 1]	$r2$.
[0.6, 1]	a :- $r3$.
[1, 1]	$r3$:- b^{G2}.
[0.6, 1]	b :- $r4$.
[1, 1]	$r4$:- a^{G1}.
[0.8, 1]	a^{G1} .
[0.8, 1]	b^{G2} .
[0.6, 1]	a^{G3} :- b^{G2}.
[0.6, 1]	b^{G4} :- a^{G1}.

m_c: $tval(\{a,b\}) \rightarrow [0,1]$

$\{m'(I+I'\_k) = m(I)*m\_c(I'\_k) \mid I'\_k$ is in $tval(\{a,b\})\}$

Fixpoint Semantics

B_P can be divided into cliques $C_1, \ldots, C_l$ such that C_k only depend on $C_1 \ldots C_{k-1}$, $1 < k \leq l$

$S_k = C_1 \cup \ldots \cup C_k$

$m_{S_k}: tval(S_k) \rightarrow [0,1]$

$S_0 = \{\}$ $\quad\quad$ $S_n = B_P$

$$m_{S_0} \Rightarrow m_{S_1} \Rightarrow m_{S_2} \Rightarrow \cdots\cdots \Rightarrow m_{S_n}$$

Model -- the belief function based on m_{S_n}

Fixpoint semantics coincides with the transformational semantics

Modular Acyclicity

- In every step, under every I being supported, there is no cycle formed by the rules involved.
  - No decyclification needed!

[0.8, 1] *a* :- *d* .
[0.8, 1] *b* :- *e* .
[0.6, 1] *a* :- *b* ∧ *f1* .
[0.6, 1] *b* :- *a* ∧ *f2* .
[1, 1] *d* .
[1, 1] *e* .
[0.5, 0.5] *f1* .
[0, 0] *f2* :- *f1* .

I1

f1	*f2*	*d*	*e*
t	**f**	**t**	**t**

$m(I1)=0.5$

[0.8, 1] *a* ~~:- *d*~~.
[0.8, 1] *b* ~~:- *e*~~.
[0.6, 1] *a* :- *b* ~~∧ *f1*~~ .
~~[0.6, 1] *b* :- *a* ∧ *f2*~~ .

I2

f1	*f2*	*d*	*e*
f	**u**	**t**	**t**

$m(I2)=0.5$

[0.8, 1] *a* ~~:- *d*~~.
[0.8, 1] *b* ~~:- *e*~~.
~~[0.6, 1] *a* :- *b* ∧ *f1*~~ .
~~[0.6, 1] *b* :- *a* ∧ *f2*~~ .

Conclusion

- Belief LP: a novel framework for reasoning with uncertainty in the presence of correlated evidence.
  - Declarative semantics
  - Operational semantics
  - Query answering algorithm
- This paper
  - extends the semantics to arbitrary belief logic programs
- Future work:
  - extend the query answering algorithm to arbitrary belief logic programs

Thank you!

Questions?