Search for More Declarativity

Backward Reasoning for Rule Languages Reconsidered

Simon Brodt   François Bry   Norbert Eisinger

Institute for Informatics, University of Munich,
Oettingenstraße 67, D-80538 München, Germany
`http://www.pms.ifi.lmu.de/`

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Rule Languages & Declarativity

Declarativity – The Greatest Advantage of Rule Languages

- Separates between
  - *What* is the problem?
  - *How* is the problem solved?
- Built-in problem-solving
  $\Rightarrow$ Allows to concentrate on problem-specification
- Add and modify rules easily
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- Finding solutions where no explicit algorithm is known
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Necessary Design Decisions

- tuple-oriented vs. set-oriented
- forward vs. backward reasoning
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No Special Assumptions for this Paper

- Complete and space-efficient search method for rule-engines
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Completeness on finite and infinite search trees.
Every node in the search space is visited after a finite number of steps.

Polynomial space complexity $O(d^c)$
$c = constant$
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Traditional Methods Fail

D-search

Incomplete on infinite trees

B-search

Exponential space-complexity in the depth of the tree

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Frequent re-evaluation

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- Use D-search
- Give rule authors some control to avoid infinite dead ends (e.g. ordering of the rules, . . .)

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Search & Declarativity

Term Representation for Natural Numbers

- `zero` represents `0`
- `succ(X,Y)` can provide the predecessor `X` to any `Y` representing a nonzero natural number

Program

```
nat(zero)  ←
nat(Y)     ←  succ(X,Y) ∧ nat(X)
nat₂(X,Y)  ←  nat(X) ∧ nat(Y)
less(X,Y)  ←  "reasonably defined"
```

Problem 1 – Incomplete Enumerations

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Queries

1. ← nat(X)
2. ← nat_2(X,Y)

Expected Results

1. Enumeration of $\mathbb{N}$
2. Enumeration of $\mathbb{N} \times \mathbb{N}$

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Queries (Assume Single-Answer-Mode)

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2. ← nat_2(X,Y) ∧ less(zero,X)

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2. Yes

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Reason – Incomplete Search

SLD-resolution is fine

Perfectly sound and complete with any literal selection function.

Problem: Incompleteness of D-search

The problems would not arise with a complete search method

Choose iterative deepening?

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odd(Y)      ← succ(X,Y) ∧ even(X)
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Query

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← constant(X) ∧ even(X)
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`constant(X)` binds `X` to some fixed, large number $n \in \mathbb{N}$.

Expected Runtime

linear, $O(n)$

Runtime with Iterative-Deepening

quadratic, $O(n^2)$

Search should not slow down the evaluation of functional rules

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- Integrates D-search and B-search
- Complete on finite and infinite trees
- Linear space complexity in depth for basic algorithm
- Non-repetitive
- Family of algorithms in parameter c with
  - Complete for $c > 0$
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Overview

1. D&B-search
2. Search & Partial Ordering
3. Conclusion

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   - The Basic Algorithm
   - The D&B-Family
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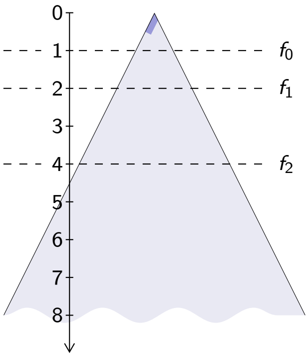

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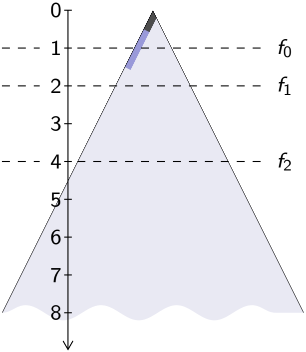

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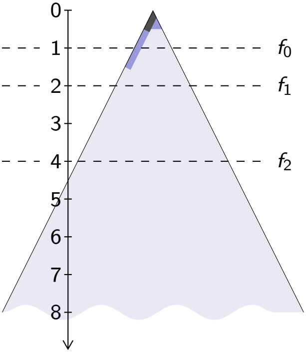

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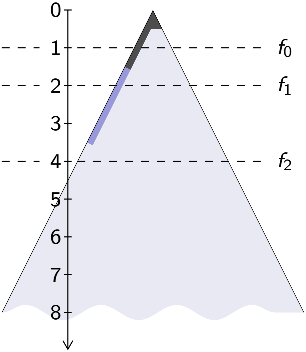

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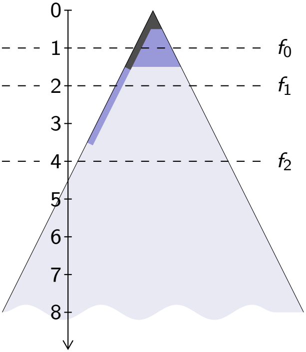

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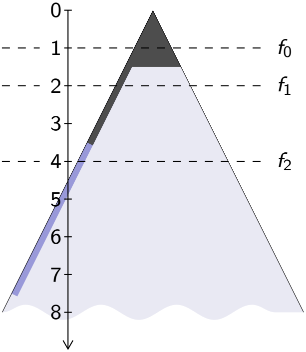

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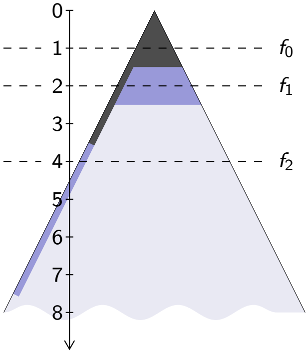

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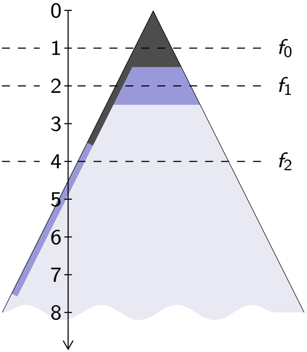

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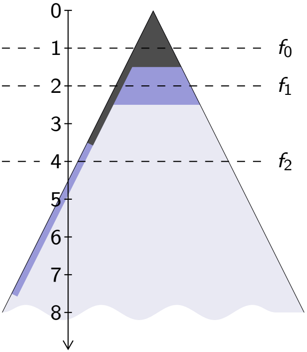
- B-search completes level i only if depth bound f_i has been passed



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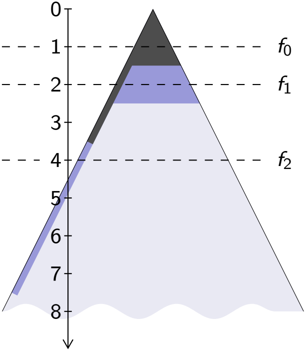

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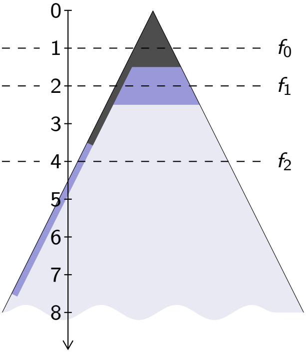

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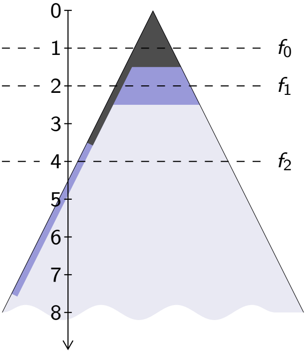
- The number of nodes to be stored is only polynomial in the maximum depth (if f_i is exponential in i)



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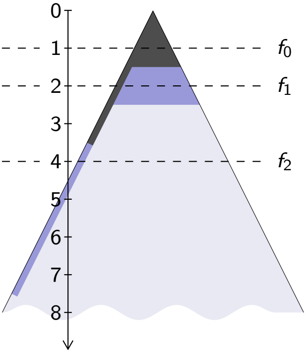

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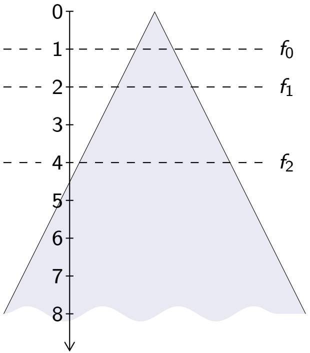
D&B-search – Idea

- Alternate D-search with B-search
- Rotation is controlled by a sequence $f_0, f_1, f_2, \ldots$ of depth bounds
  - Defined by a function $\mathbb{N} \rightarrow \mathbb{N}$, $i \mapsto f_i$
  - $i < f_i < f_{i+1}$

$f_i = 2^i$ for the examples

D&B-search – Pivot-Nodes and Pivot-Sets



A node is "earlier" than another if (unrestricted) D-search would expand it first

- *Pivot-node* s_i : earliest node at depth f_i
- *Pre-pivot-set* S_0 : nodes earlier than s_0
- D_i : nodes earlier than s_{i+1}
- B_i : nodes at depth i
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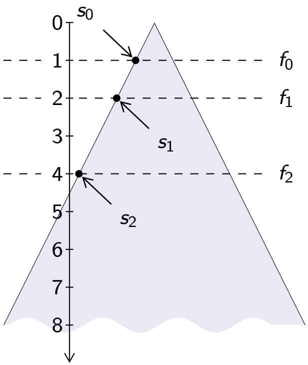
- *Post-pivot-set* R : the rest of the nodes

D&B-search – Pivot-Nodes and Pivot-Sets



A node is "earlier" than another if (unrestricted) D-search would expand it first

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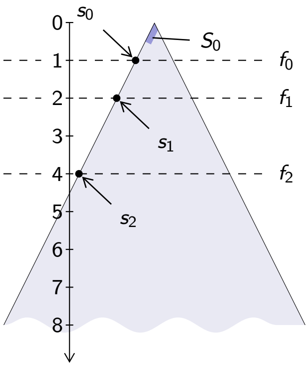

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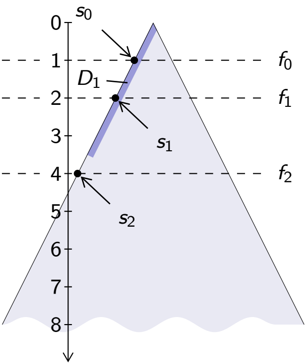

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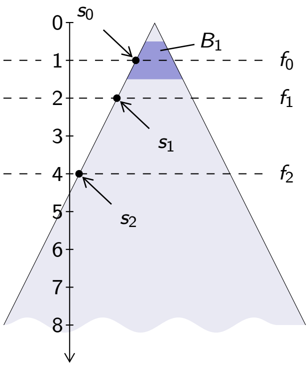

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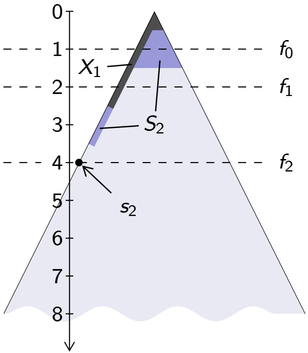

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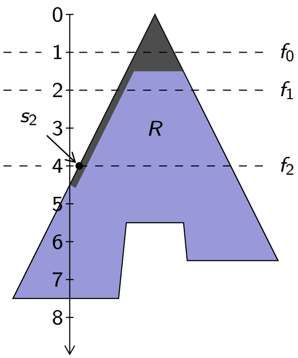

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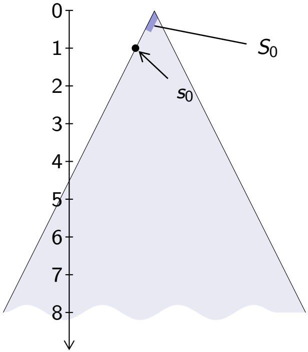
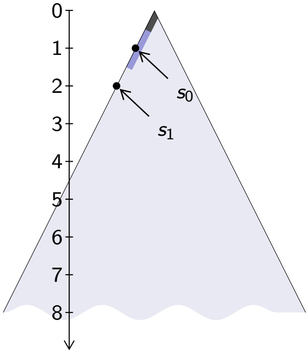
D&B-search – Complete Infinite Tree



- D-search expands all nodes in S_0
- D-search passes s_0
- S_1 is finished
- D-search passes s_1
- B-search expands the rest of B_1
- S_2 is finished
- D-search passes s_2
- B-search expands the rest of B_2
- S_3 is finished

D&B-search – Complete Infinite Tree



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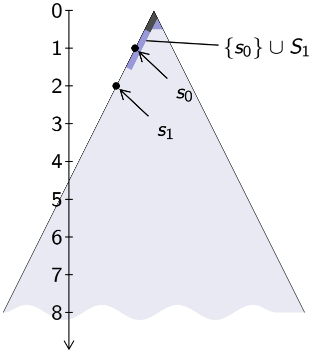

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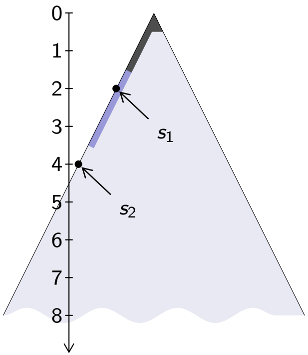

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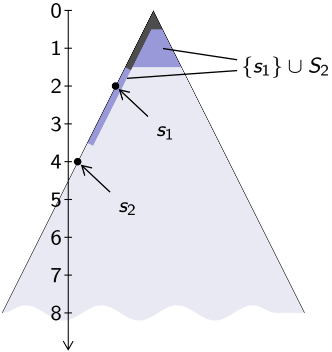

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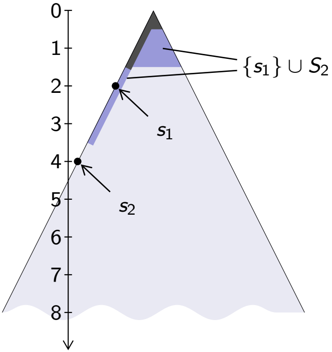

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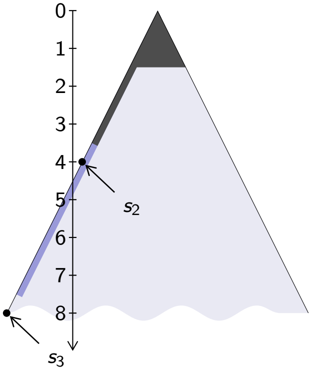

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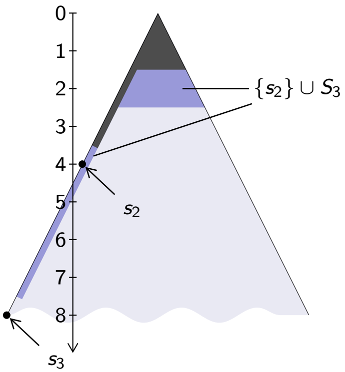

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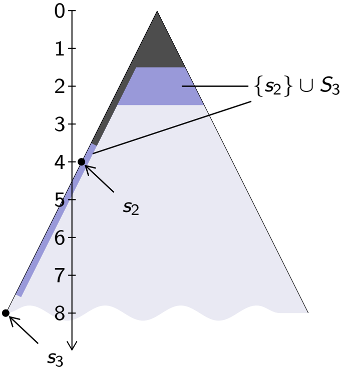

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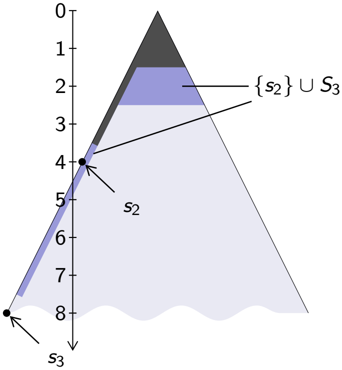

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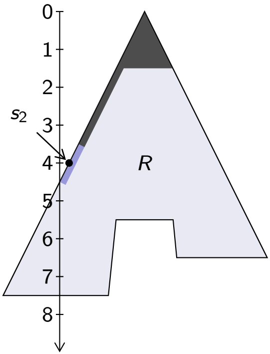
- S_2 is finished
- D-search passes s_2
- B-search expands the rest of B_2
- S_3 is finished

Observation

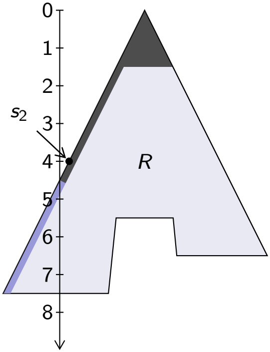
D&B-search expands the nodes in the order
$S_0, s_0, S_1, s_1, \ldots, S_i, s_i, \ldots, R$

D&B-search – Finite Tree



- S_2 is finished
- D-search expands s_2
- D-search reaches the max. depth in R (no s_3 in this tree)
- B-search may complete B_2
- D-search continues R
- D-search continues R
- D-search finishes R
- Search is finished

D&B-search – Finite Tree



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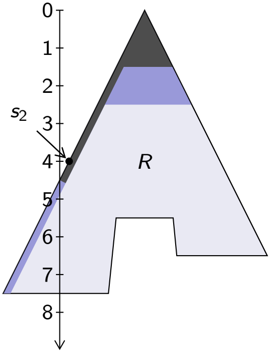

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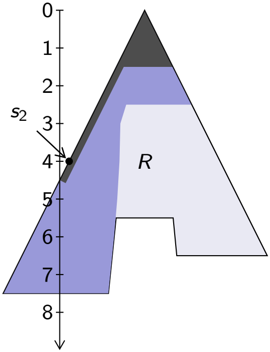

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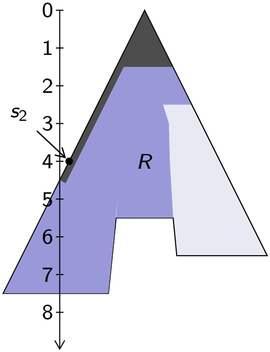

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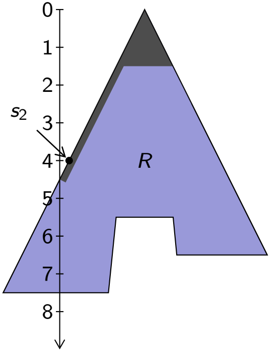

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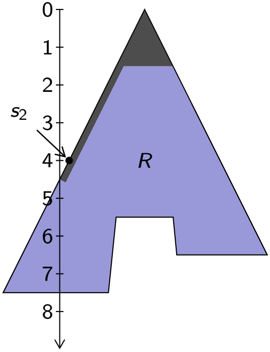

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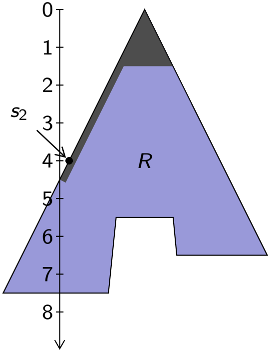
Observation

- B-search stops shortly after D-search reaches the max. depth
- Most of the tree is expanded by D-search

D&B-search – Finite Tree



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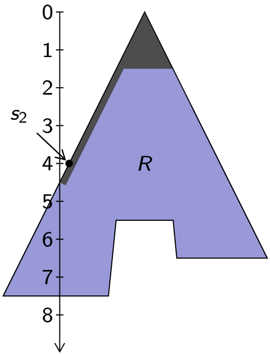

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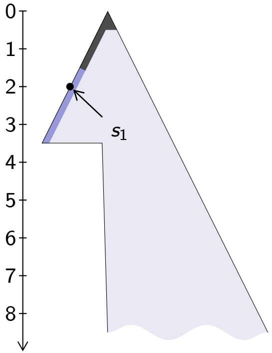
D&B-search – Non-Complete Infinite Tree



- S_1 is finished
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- D-search finishes S_2
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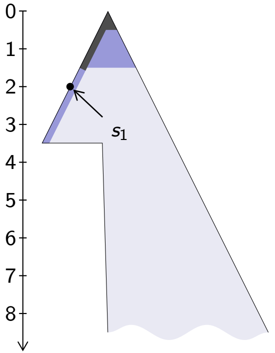
D&B-search – Non-Complete Infinite Tree



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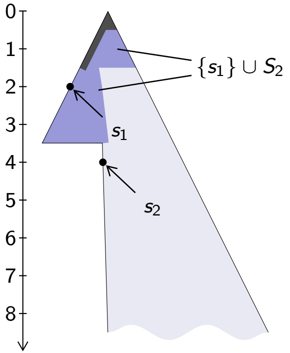

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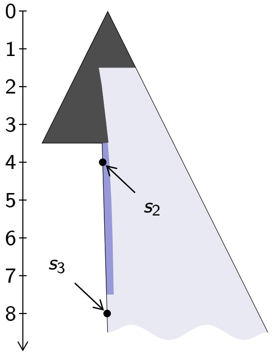

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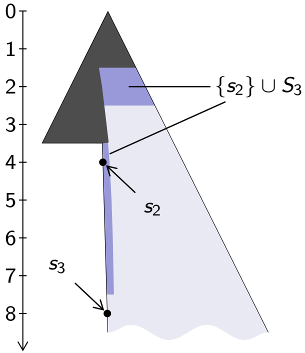

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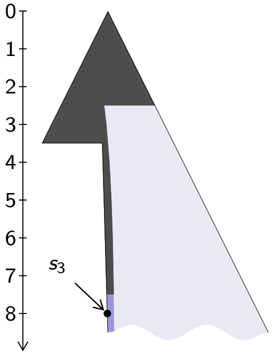

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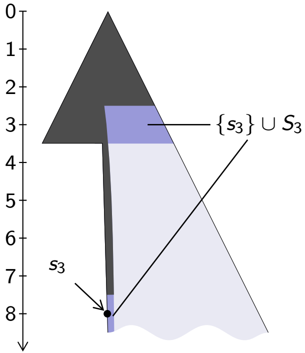

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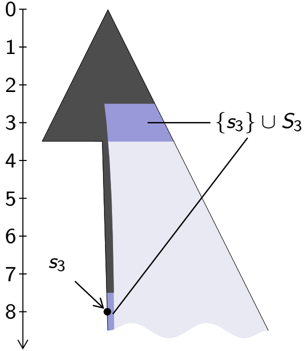
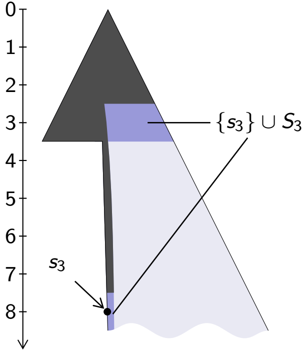


- S_1 is finished
- D-search passes s_1
- B-search may complete B_1
- D-search finishes S_2
- D-search passes s_2
- B-search completes B_2
- S_3 is finished
- D-search passes s_3
- B-search completes B_3

Observation

- D-search “vanishes” in the earliest infinite branch
- Most of the tree is expanded by B-search

D&B-search – Non-Complete Infinite Tree



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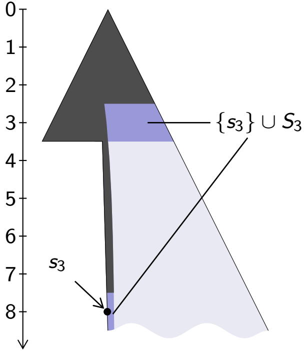

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D&B-search – Adaptivity

D&B-Search

- behaves almost like D-search on finite trees
- behaves almost like B-search on infinite trees

$\Rightarrow$ **has a kind of built-in adaptivity**

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The D&B-Family

Assume that the tree's branching factor is bounded by $b \in \mathbb{N}$

- Parameterise the function f_i with $c \in \mathbb{N} \cup \{\infty\}$
- Idea: $f_{c,i} := \lfloor b^{\frac{i}{c}} \rfloor$
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Properties

- For $1 \leq c \leq \infty$ the algorithm is complete (for $c = 0$ it is not)
- For $1 \leq c < \infty$ its space complexity is $O(d^c)$
- For $c = 0$ it corresponds to D-search because $f_{0,0} = \infty$. The pre-pivot-set S_0 contains all nodes of the whole tree.
- For $c = \infty$ it corresponds to B-search because $f_{\infty,i} = i + 1$. All sets $D_i \backslash X_i$ are empty, thus $S_{i+1} = B_i \backslash \{s_i\}$

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- Transforms problems on search algorithms to problems on partial orderings
- Idea: Nodes ordered by their first occurrence
- Partial orderings are a well-studied field
  - precise notation
  - Powerful instruments for proofs (e.g. the arithmetic for ordinal numbers)
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Characterization of Completeness

A search algorithm is complete iff for each depth i there is a depth $f_{i+1} > i$ so that none of the nodes at depth f_{i+1} is expanded before every node at depth i has been expanded.

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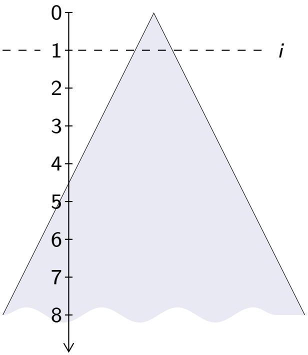

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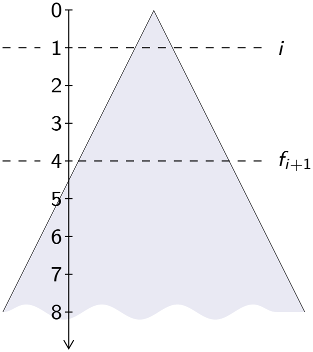

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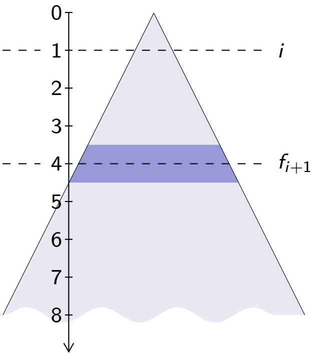

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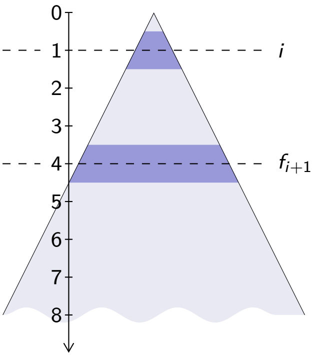

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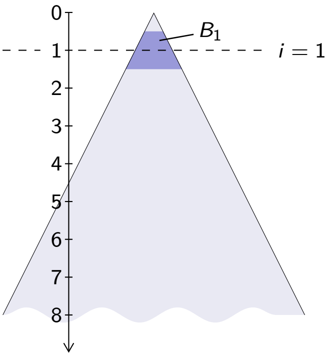

D&B-search

- $B_i \subseteq S_{i+1} \cup X_i$
- $S_{i+1} \cup X_i$ is completed before s_i, the first node at depth f_{i+1}

$\Rightarrow$ D&B-search is complete

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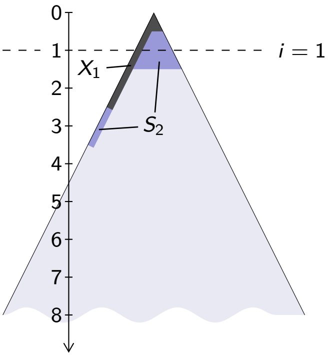

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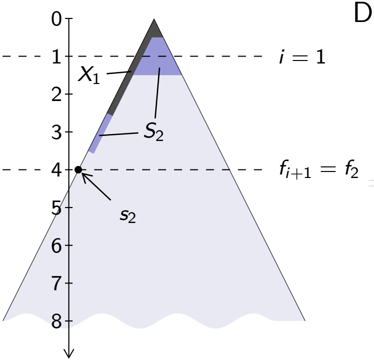

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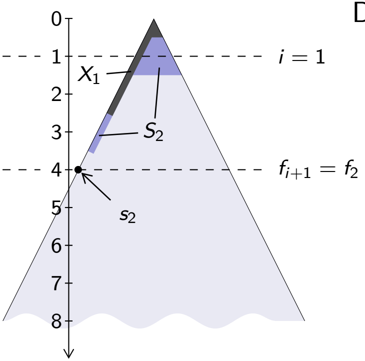

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Conclusion

1. D&B-search
2. Search & Partial Ordering
3. Conclusion

D&B-search

- Novel Search method: Integrating D-search and B-search
- Ratio of D-search and B-search balanced by a parameter
- Family of algorithms in parameter c
  - D-search and B-Search as borderline cases
  - Complete in all non-borderline cases
  - Non-repetitive, i.e. time complexity is linear in size
  - Space complexity is polynomial in depth.
    Polynomial depends on parameter c
- Formal proofs of these properties
- Built-in adaption to the searched tree
- Better than running D-Search and B-Search in parallel
- Implementation in form of detailed pseudo-code
  $\longrightarrow$ only simple datastructures needed

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Theoretical-Framework

- Based on partial orderings
- Covers finite and infinite trees uniformly
- High analytic power, concise and precise proofs

Future work

- Combine D-search and iterative deepening to D&I-search by the same principle
  - Behaves (almost) like D-search on finite trees
  - Behaves (almost) like iterative-deepening on infinite trees
  - Achieved by the same depth bounds f_i as for D&B-search
- Same for other combinations
- Prototype implementation
- Empirical comparison to other uninformed search methods
  $\longrightarrow$ Focus: Logic programming applications using backward reasoning approaches with and without memorization

Thank You